# Package 'mosaicCalc'

July 26, 2024

Type Package

Title R-Language Based Calculus Operations for Teaching

Description Software to support the introductory \*MOSAIC Calculus\* textbook <<https://www.mosaic-web.org/MOSAIC-Calculus/>>), one of many data- and modeling-oriented educational resources developed by Project MOSAIC (<<https://www.mosaic-web.org/>>). Provides symbolic and numerical differentiation and integration, as well as support for applied linear algebra (for data science), and differential equations/dynamics. Includes grammar-of-graphics-based functions for drawing vector fields, trajectories, etc. The software is suitable for general use, but intended mainly for teaching calculus.

Version 0.6.4

**Depends**  $R (= 3.5.0)$ 

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Aquaman *Box office from the movie Aquaman*

#### Description

Earnings each weekend after the release of the movie *Aquaman*

#### Usage

Aquaman

## Format

A data frame with 20 rows and variables Date, Earnings (in USD), and the count of the Weekend.

#### Source

Beth Schaubroeck at USAFA. Scraped from [https://www.boxofficemojo.com/release/rl31088](https://www.boxofficemojo.com/release/rl3108800001/weekend/?ref_=bo_rl_tab)00001/ [weekend/?ref\\_=bo\\_rl\\_tab](https://www.boxofficemojo.com/release/rl3108800001/weekend/?ref_=bo_rl_tab).

<span id="page-3-0"></span>

#### Description

Find local extreme points

#### Usage

argM(tilde, domain)

#### Arguments



#### Details

End-points of the domain may be included in the output.

#### Value

A data frame with values for x, the function output at those values of x, and the concavity.

#### Examples

 $argM(x^2 \sim x, domain(x=-1:1))$ 

basis\_sets *Basis sets for for function approximation*

#### Description

These functions generate the mathematical functions for three different basis sets: Fourier (sines), Legendre (orthogonal polynomials), and Splines (low-order smooth approximation)

#### Usage

```
legendre_set(df = 3, left = -1, right = 1)
legendre_M(x, df, left = -1, right = 1)
ns_set(df = 3, left = -1, right = 1)fourier_M(x, n, fperiod = NULL)
```
<span id="page-4-0"></span> $ns_M(x, df = NULL, knots = NULL, intercept = FALSE, Boundary.knots = range(x))$ 

fourier\_set(df, left =  $-1$ , right = 1)

#### Arguments



#### Details

For each basis, there are two different forms for the generating functions. Names ending in \_set create a set of functions with arguments x and n, where integer n provides an index into the set. The same names with a  $_M$  suffix produce a model matrix corresponding to a specified set of x values. These are useful with  $lm()$  and similar model-building functions in the same way that  $poly()$  and ns() are useful. (ns\_M() is just an alias for splines::ns().) Like poly() and ns(), the \_M suffix functions do *NOT* include an intercept column.

#### Value

The \_M functions return a model matrix. The \_set functions return a function with arguments x and n. The integer n specifies which function to use, while  $x$  is the set of values at which to evaluate that function.

Blob1 *Shapes used in moment of inertia calculations*

#### Description

These are used in a handful of examples in the *MOSAIC Calculus* text.

#### **6** bounds **bounds bounds bounds bounds bounds bounds bounds**

#### Usage

Blob1 Blob2 Blob3 Blob4

#### Format

Data frames with coordinates x and y as columns

An object of class tbl\_df (inherits from tbl, data.frame) with 200 rows and 2 columns. An object of class tbl\_df (inherits from tbl, data.frame) with 200 rows and 2 columns. An object of class tbl\_df (inherits from tbl, data.frame) with 200 rows and 2 columns.

Body\_fat *Measurements of body-fat percentage and related quantities*

#### Description

It's hard to measure body fat. This data frame records a direct measure of body-fat percentage along with easier-to-make measurements. The question of interest in collecting the data is whether there is a way to estimate body-fat percentage from the easier-to-measure quantities.

#### Usage

Body\_fat

#### Format

An object of class spec\_tbl\_df (inherits from tbl\_df, tbl, data.frame) with 252 rows and 14 columns.

bounds *Specify a domain over which a function is to be graphed*

#### <span id="page-5-1"></span>Description

domain() is used with slice\_plot(), contour\_plot(), or interactive\_plot() to describe the plotting region. There is a standard syntax for domain() (see the first example) but there are also shortcuts. In the shorthand syntaxes, you can but don't have to specify the name of the input. If it's not specified, the plotting programs will try to do something sensible. But better to specify the names explicitly.

<span id="page-5-0"></span>

<span id="page-6-0"></span>box\_set 7

#### Usage

```
bounds(...)
```
domain(...)

#### Arguments

... One or more expression of the form x = -5:5

#### Details

The colon operator is masked so that, for instance,  $x = 0.5:1.3$  literally means "0.5 to 1.3", and not just 0.5 as the base colon operator would give.

#### Value

a list with one component for each element in . . .

#### Examples

```
## Not run:
contour_plot(x / y ~ x + y, bounds(x=-5:5, y=1:4))
slice_plot(x^2 ~ x, bounds(x = 2.5:4.2)) # overrides colon operator
slice_plot(x^2 ~ x, bounds(x = c(2.5, 4.2)))
slice_plot(x^2 ~ x, bounds(x = 1 %pm% 0.5))
## End(Not run)
```
box\_set *Evenly spaced samples across a one- or two-dim domain*

#### Description

This function breaks up a domain in 1- or 2- dimensions into evenly spaced samples. It returns a data frame of the position of the samples, each of which can be considered to correspond to a Riemann bin for the purposes of integration.

#### Usage

```
box_set(tilde = NULL, domain, n = 10, sum = FALSE, dx = NULL)
```
#### <span id="page-7-0"></span>Arguments



#### Value

By default, a data frame listing the location of the samples, the . output. value of the given function at those samples, the spatial extent of the samples (that is, dx for one-dimension or dx and dy for two dimensions. There is also a dA giving the dx or dx\*dy depending on the dimension). If sum=TRUE, then returns the sum of . output  $*$  dA, that is, the estimate of the integral of the function over the domain.

#### Examples

```
box_set(x*y ~ x & y, domain(x=0:1, y=0:1), n = 4)
# approximation to the variance of a uniform [0,1] distribution
box_set((x-,5)^2 ~ x, domain(x=0:1), n=100, sum=TRUE)
# a polygon
poly \leq tibble(x = c(1:9, 8:1), y = c(1, 2\times(5:3), 2, -1, 17, 9, 8, 2:9))
boxes \leq box_set(1 \sim x & y, poly, dx = 1)
gf_polygon(y ~ x, data = poly, color="blue", fill="blue", alpha=0.2) %>%
  gf_rect((y - dy/3) + (y + dy/3) ~ (x - dx/3) + (x + dx/3),data = boxes)# area inside polygon
box_set(1 \sim x & y, poly, n=100)
```
Boyle *Robert Boyle's pressure vs volume measurements*

#### **Description**

Pressure versus volume of air at a constant temperature as collected by Robert Boyle. Units are arbitrary.

#### Usage

Boyle

#### Format

A data frame with 7 rows and numerical variables pressure and volume.

#### <span id="page-8-0"></span>Cello 9

#### Details

Robert Boyle (1627-1691) was a founder of modern chemistry and the scientific method in general. As any chemistry student already knows, Boyle sought to understand the properties of gasses. His results are summarized in *Boyle's Law*. The data frame Boyle contains two variables from one of Boyle's experiments as reported in his lab notebook: pressure in a bag of air and volume of the bag. The units of pressure are mmHg and the units of volume are cubic inches.

#### Source

Boyle's notebooks are preserved at the Royal Society in London. The data in the Boyle data frame have been copied from https://chem.libretexts.org/Courses/University\_of\_California\_Davis/UCD\_Chem\_002A/

Cello *Short recordings of a cello and a violin*

#### Description

Only one note is involved.

- Cello: a complete note
- Violin: a complete note
- Cello\_seg: a brief snipet of sound after the initial transient
- Violin\_seg: as in Cello\_seg

#### Usage

Cello

Cello\_seg

Violin

Violin\_seg

#### Format

Data frames (for convenience) with many rows and these variables:

- y the amplitude of the sound wave (arbitrary units)
- t the time of the sample in seconds.

An object of class tbl\_df (inherits from tbl, data. frame) with 3616 rows and 2 columns. An object of class tbl\_df (inherits from tbl, data.frame) with 43776 rows and 2 columns. An object of class tbl\_df (inherits from tbl, data. frame) with 3638 rows and 2 columns.

#### <span id="page-9-0"></span>Description

The rate at which crickets chirp is related to the ambient temperature. These data are motivated and described at [https://www.globe.gov/explore-science/scientists-blog/archived-posts/](https://www.globe.gov/explore-science/scientists-blog/archived-posts/sciblog/index.html_p=45.html) [sciblog/index.html\\_p=45.html](https://www.globe.gov/explore-science/scientists-blog/archived-posts/sciblog/index.html_p=45.html)

#### Usage

data(Chirps)

#### Format

A data frame with 55 rows

- date Which year the estimate is for.
- time in 24-hour format
- chirps the number of chirps per 15 seconds
- temperature in degrees F

contour\_plot *Contour plots of functions of two variables*

#### Description

Creates a ggplot2-compatible contour plot of a function of two variables.

#### Usage

```
contour_plot(
  ...,
 npts = 100,
 labels = TRUE,
  filed = TRUE,contours_at = NULL,
  n_{\text{contours}} = 10,
  n_fill = 50,
  alpha = 1,
  fill\_alpha = 0.1,
  label_1alpha = 1,
  label_placement = 0.5,
  contour_color = "blue",
  label_color = contour_color,
```
#### <span id="page-10-0"></span>contour\_plot 11

```
skip = 1,guide = FALSE,
 guide_title = "output",
 color_scale = scale_color_viridis_c(),
 fill_scale = scale_fill_viridis_d()
\mathcal{L}
```
#### Arguments



#### Value

ggplot2 graphics layers

### Examples

```
## Not run:
contour_plot(sin(0.2*x*y) ~ x & y, domain(x=-10.3:4.5, y=2:7.5))
contour\_plot(x + y ~ \sim x ~ 8 y)contour_plot(sin(0.2*x*y)) # but better to use tilde expression
## End(Not run)
```
<span id="page-11-0"></span>

#### Description

For documentation, see https://covid19datahub.io/articles/doc/data.html

#### Usage

Covid\_US

#### Format

An object of class grouped\_df (inherits from tbl\_df, tbl, data.frame) with 575 rows and 36 columns.

#### Details

See Guidotti, E., Ardia, D., (2020), "COVID-19 Data Hub", Journal of Open Source Software 5(51):2376, doi: 10.21105/joss.02376.



#### Description

Since single-chip central processing units were introduced in the early 1970s, performance has increased exponentially. Around 1965, Gorden E. Moore, a co-founder of the integrated circuit giant Intel, postulated that capacity of chips, measured as the number of transistors, doubles approximately every two years. Known as Moore's Law, this pattern has held true for decades.

#### Usage

data(CPUs)

#### Format

A data frame with 128 cases, each a computer CPU.

- processor Model name/number of the processor
- transistors Number of transistors
- year Year of market introduction of the CPU
- designer The company that introduced the CPU
- process The width of circuit elements, in nm.
- area Area of the chip in mm-squared.
- density The number of transistors per unit area, simply transistors/area.

<span id="page-12-0"></span>create\_num\_antiD 13

#### Source

Dr. Joe Eichholz



#### Description

This will typically be called directly from antiD() when an integral can't be handled symbolically.

#### Usage

 $create\_num\_antilOfilde, ..., lower = NULL, .tol = 1e-04)$ 

#### Arguments



#### Value

a function with the w.r.t. variable as the first argument. The function is a wrapper around numerical integration routines.

<span id="page-12-1"></span>

D *Derivative and Anti-derivative operators*

#### <span id="page-12-2"></span>Description

Operators for computing derivatives and anti-derivatives as functions.

#### Usage

```
D(tilde, ...)
antiD(tilde, ..., lower.bound = 0, force.numeric = FALSE, .tol = 1e-04)
```


#### Details

D() attempts to find a symbolic derivative for simple expressions, but will provide a function that is a numerical derivative if the attempt at symbolic differentiation is unsuccessful. The symbolic derivative can be of any order (although the expression may become unmanageably complex). The numerical derivative is limited to first or second-order partial derivatives (including mixed partials). antiD() will attempt simple symbolic integration but if it fails it will return a numerically-based anti-derivative.

antiD() returns a function with the same arguments as the expression passed to it. The returned function is the anti-derivative of the expression, e.g., anti $D(f(x) \sim x)$  ->  $F(x)$ . To calculate the integral of  $f(x)$ , use  $F(to) - F(from)$ .

#### Value

For derivatives, the return value is a function of the variable(s) of differentiation, as well as any other symbols used in the expression. Thus,  $D(A*x^2 + B*y \sim x + y)$  will compute the mixed partial with respect to x then y (that is,  $\frac{d^2f}{dy dx}$ ). The returned value will be a function of x and y, as well as A and B. In evaluating the returned function, it's best to use the named form of arguments, to ensure the order is correct.

a function of the same arguments as the original expression with a constant of integration set to zero by default, named "C", "D", ... depending on the first such letter not otherwise in the argument list.

#### Examples

```
D(sin(t) \sim t)D(A*sin(t) \sim t)D(A * sin(2 * pi * t / P) ~ t, A=2, P=10) # default values for parameters.
f \leq D(A*x^3 \sim x + x, A=1) # 2nd order partial -- note, it's a function of x
f(x=2)f(x=2,A=10) # override default value of parameter A
g \leftarrow D(f(x=t, A=1)^2 \sim t) # note: it's a function of t
```
#### <span id="page-14-0"></span>df2matrix 15

```
g(t=1)gg \leftarrow D(f(x=t, A=B)^2 \sim t, B=10) # note: it's a function of t and B
gg(t=1)gg(t=1, B=100)
f <- makeFun(x^2^x)D(f(cos(z))\sim z) #will look in user functions also
antiD(a*x^2 \sim x, a = 3)
G \le antiD(A/x \sim x) # there will be an unbound parameter in G()
G(2, A=1) # Need to bound parameter. G(2) will produce an error.
F <- antiD( A*exp(-k*t^2) \sim t, A=1, k=0.1)
F(t=Inf)
one = makeFun(1 \sim x)
by.x = antib(one(x) \sim x)by.xy = antiD(by.x(sqrt(1-y^2)) ~ y)
4 * by.xy(y = 1) # area of quarter circle
```
df2matrix *Construct a model matrix from data as if by hand*

#### Description

The *MOSAIC Calculus* course includes a block on linear algebra. As part of this block, we want to cover creating model matrices and evaluating the model constructed from them. One way to do this is to introduce  $lm()$  and the domain specific language for specifying model terms. However, that introduces oddities. For instance,  $\text{lm}(mpg \sim hp + hp^2)$ , data = mtcars) does NOT add the quadratic term to the model matrix. Also, lm() doesn't produce a residual or the model vector (ahem ... the fitted values) in the form of vectors. That's fine if you're teaching statistical modeling, but in the *MOSAIC Calculus* linear algebra block we are not teaching statistics, but the mathematical pre-requisites to understanding statistics.

#### Usage

df2matrix(..., data = NULL)

#### Arguments



#### Details

Specifically for *MOSAIC Calculus*, we have added this df2matrix() function. It serves much the same purpose as cbind(), that is, collecting vectors into a matrix. But is has two additional features:

1. It has a data= argument so that it can refer to a data frame.

2. It names the columns of the matrix with the code that was used to create each column.

<span id="page-15-0"></span>The consequence of (2) is that the x vector produced by  $qr$ . solve() will have the same names as the matrix. That helps in interpreting x. But those names can also be used by makeFun() to generate a function from x.

1 is a good form in which to write the intercept term.

#### Value

a matrix

#### Examples

```
A <- df2matrix(1, disp, log(hp), sin(cyl)*sqrt(hp), data = mtcars)
b \leq -df2matrix(mpg, data = mtcars)
x \leftarrow qr.solve(A, b)f \leftarrow \text{makeFun}(x)f(hp=3, \text{disp=2, cyl=4})
```
EbolaAll *Case numbers in an Ebola outbreak in 2014*

#### Description

In December 2013, an 18-month-old boy from a village in Guinea suffered fatal diarrhea. Over the next months a broader outbreak was discovered, and in mid-March 2014, the Pasteur Institute in France confirmed the illness as Ebola-Virus Disease caused by the Zaire ebolavirus. Although the outbreak was first recognized in Guinea, it eventually encompassed Liberia and Sierra Leone as well. By July 2014, the outbreak spread to the capitals of all three countries.

#### Usage

EbolaAll

EbolaGuinea

#### Format

A dataframe with 182 rows. Each row is a daily report during a period of over 18 months during 2014 and 2015. Each report gives the number of new cases and disease-related deaths since the last report in each or three countries: Sierra Leon, Liberia, and Guinea. These values have been calculated from the raw, cumulative data. The data have been scrubbed to remove obvious errors.

- Date: Date when the World Health Organization issued the report
- Gcases: Number of new cases in Guinea
- Gdeaths: Number of new deaths in Guinea
- Lcases: Number of new cases in Liberia
- Ldeaths: Number of new deaths in Liberia
- <span id="page-16-0"></span>• SLcases: Number of new cases in Sierra Leone
- SLdeaths: Number of new deaths in Sierra Leone
- TotCases: Cumulative number of cases across all three countries
- TotDeaths: Cumulative number of deaths across all three countries Added in EbolaGuinea
- Days: When the report was issued in terms of a count of days from the initial report.
- G7Rcases: Number of new cases in Guinea averaged across 7 reports
- G7Rdeaths: Number of new deaths in Guinea averaged across 7 reports

An object of class spec\_tbl\_df (inherits from tbl\_df, tbl, data.frame) with 182 rows and 6 columns.

#### Details

- EbolaAll gives daily reports for Guinea, Liberia, and Sierra Leone.
- EbolaGuinea contains just the Guinea data, but adds columns containing a 7-day moving average.

#### Source

US Centers for Disease Control (CDC).

Effective\_oxygen *Effective amount of oxygen available at different altitudes*

#### Description

Air pressure changes with altitude and therefore the amount of oxygen available changes as well.

#### Usage

```
data(Effective_oxygen)
```
#### Format

A data frame with 30 rows

- altitude' In feet above sea level
- oxygen' Effective oxygen as a proportion of ai-

#### Source

<https://wildsafe.org/resources/ask-the-experts/altitude-safety-101/oxygen-levels/>

#### <span id="page-17-0"></span>Description

Internal combustion engines have been built for a variety of purposes ranging from propelling small model airplanes to powering trucks to driving the propellers of giant ships. This data frame gives the characteristics of a wide size range of such engines.

#### Usage

data(Engines)

#### Format

A data frame with 39 cases, each of which is an internal combustion engine, with observations on the following variables.

- mass In pounds
- ncylinder Number of cylinders
- stroke The length of the stroke made by a piston
- strokes Whether the engine cycle is 2-stroke or 4-stroke.
- displacement In cc
- bore Diameter of the piston
- BHP The power generated by the engine. One BHP is the same as 745.7 Watts.
- RPM The speed, in revolutions per second, at which the engine generates the listed BHP.

#### Source

McMahon, Thomas A., and John Tyler Bonner. On Size and Life. New York: Scientific American Library, 1983. pp. 60-61

first\_three\_args *Handle the first three arguments of graphics functions*

#### Description

This function is intended for package developers, not end-users. Canonically, mosaicCalc functions that produce layerable graphics have three initial arguments in a specific order: (1) a previous gg layer,  $(2)$  a tilde expression, and  $(3)$  a domain. But either  $(1)$  or  $(3)$  can be missing. first\_three\_args() translates a leading ... argument into the list of the canonical three initial arguments, returning them as components of a list. In addition, there may be additional arguments in ... that specify other aspects of the plot, e.g. color.

#### <span id="page-18-0"></span>fitSpline that the state of the state of

#### Usage

first\_three\_args(..., two\_tildes = FALSE)

#### Arguments



#### Details

In constructing a mosaicCalc graphics layer, the function (e.g. slice\_plot() or contour\_plot()) whould have ... as its first argument. Intercept that ... with first\_three\_args() to extract the first three canonical arguments as components gg, tilde, and domain of a list. Any remaining arguments in ... will be placed in the dots component.

<span id="page-18-1"></span>fitSpline *Find zeros of a function*

#### Description

This is defined in the mosaic package: See [fitSpline](#page-18-1)

Fly\_ball *Trajectory of a fly ball in baseball*

#### Description

The trajectory of a fly ball as calculated by a sophisticated model involving drag, spin, and such. The physics and mathematics are described at [http://baseball.physics.illinois.edu/Trajectory](http://baseball.physics.illinois.edu/TrajectoryAnalysis.pdf)Analysis. [pdf](http://baseball.physics.illinois.edu/TrajectoryAnalysis.pdf)

#### Usage

Fly\_ball

#### Format

A data frame with 552 rows.

- t (seconds). Runs from 0 to 5.51 s
- y horizontal position (feet)
- z vertical position (feet)

#### Details

The ball has a mass of 5.125 ounces and a circumference of 9.125 inches The speed off the bat was 103 mph at a launch angle of 27.5 degrees from the horizontal. The ball was hit with backspin at 2500 rpm by a right-handed batter. Ambient temperature 70 deg F with a relative humidity of 50% and a barometric pressure of 29.92 inHg. The field was at an elevation of 15 feet.

The position of the ball was recorded every 0.01 seconds, for 5.5 seconds altogether. If you prefer to work with continuous functions of time, you can construct them. See the examples.

#### Source

```
http://baseball.physics.illinois.edu/TrajectoryCalculator-new-3D.xlsx
```
#### Examples

```
# Constructing continuous functions of time
ball_z <- mosaic::connector(z \sim t, data = Fly_ball)
ball_y \leftarrow mosaic::connector(y \sim t, data = Fly_ball)
```
gradient\_plot *Plot a vector field*

#### Description

Plot a vector field

#### Usage

```
gradient\_plot(..., npts = 20, color = "black", alpha = 0.5, transform = sqrt)vectorfield_plot(
  ...,
 npts = 20,
  color = "black",
  alpha = 0.5,
  transform = sqrt,
  env = NULL)
```
#### Arguments



<span id="page-19-0"></span>

<span id="page-20-0"></span>

#### Details

There will be one tilde expression for the horizontal component of the vector and another tilde expression for the vertical component. Suppose the horizontal axis is called u and the vertical axis is called v, as would be established by a bounds specification like bounds ( $u=-1:1$ ,  $v=-1:1$ ). Then the horizontal tilde expression must have a left side called  $u \sim$ . Similarly, the vertical tilde expression will have a left side called  $v \sim$ . On the right side of the tilde expressions will go the formulas for the respective components of the vectors, e.g.  $u \sim \sin(u-v)$  and  $v \sim v \star u^2$ .

Typically, the length of the arrows is not meaningful in the units of the horizontal or vertical axis. For instance, in a gradient plot of  $f(x,y)$ , the axis is in units of x, but the gradient component has units of  $f(x,y)/x$ . Similarly for the flow of a differential equation. Nonetheless, the relative lengths of the arrows, one to another, does have meaning. In drawing the vector field, the arrows are scaled so that the longest one barely avoids contact with it's neighbors. This natural scaling has the disadvantage that it can be hard to discern the lengths of the shortest arrows, which often are near zero (as with the gradient near the argmax or argmin, or near a fixed point of a differential equation flow field). By default, the relative lengths of the arrow are transformed by sqrt, to make the long arrows shorter and therefore enable the sort arrows to be drawn somewhat longer. If you want the natural scaling instead, use transform=I. Or you might want to make the arrows even more similar in length. Then use, for instance transform=function(L) L^0.1

#### Value

ggplot2 graphics layers

#### Examples

```
gradient_plot(x * sin(y) * x & y, bounds(x=-1:1, y=-1:1), transform=I)
vectorfield_plot(x ~ -y, y ~ x, bounds(x=-1:1, y=-1:1))
gf_label(0 ~ 0, label="center", color="red") %>%
vectorfield_plot(x ~ -y, y ~ x, bounds(x=-1:1, y=-1:1), transform=function(x) x^0.2 )
vectorfield_plot(u ~ sin(u-v), v - v * u^2, bounds(u=0:1, v=-1:1))
```
HDD\_Minneapolis *Heating degree days in Minneapolis, Minnesota, USA*

#### Description

A "heating degree day" is a measure of weather coldness. It's defined to be the difference between the outdoor ambient temperature and 65 degrees F, but has a value of zero when the ambient temperature is above 65 degrees. This difference is averaged over time and multiplied by the number of days in the time period covered. The heating degree day is often used as a measure of the demand for domestic heating in a locale.

#### <span id="page-21-0"></span>Usage

```
data(HDD_Minneapolis)
```
#### Format

A data frame HDD\_Minneapolis with 1412 rows and 4 variables:

- year the year
- month the month
- hdd the number of heating degree days for that period.
- loc the location at which the temperature was measured. In the early years, this was downtown Minneapolis. Later, the site was moved to the Minneapolis/Saint-Paul International Airport.

#### Details

These data report monthly heating degree days. For teaching purposes, the data give an extreme example of how a relationship (hdd vs year) can be revealed by including a covariate (month). Although interest focusses on the change in temperature over the century the data cover, there is such regular seasonal variation that no systematic trend over the years is evident unless month is taken into account.

#### Examples

```
mod_1 <- lm(hdd \sim year, data = HDD_Minneapolis)mod_2 < -1m(hdd \sim year + month, data = HDD_Minneapolis)
```
Home\_utilities *Gas and electricity usage by a home in St. Paul, MN*

#### Description

Gas and electricity usage by a home in St. Paul, MN

#### Usage

```
data(Home_utilities)
```
#### Format

A data.frame object with one row for each month

- month the month covered by the bill as a number
- day the terminating day of the bill
- year the year in which the month fell
- temp average temperature over the entire month
- kwh electricity usage in kilowatt-hours
- <span id="page-22-0"></span>• ccf - natural gas usage in cubic feet
- thermsPerDay another measure of natural gas usage
- dur duration of the period covered by the bill, in days (typically about 30)
- totalbill dollar amount of the bill
- gasbill: dollar amount of the natural gas part of the bill
- elecbill dollar amount of the electricity bill
- generatedKwh Energy generated by the photovoltaic system
- notes miscellaneous comments on the month's bill or changes in the house

#### Details

A college mathematics professor teaching in St. Paul, Minnesota, USA collected these records of his utility bills starting in 1999. Starting in June 2022, solar photovoltaics were installed on the roof of the house. There are months missing, a few transcription errors, etc. **These data will be** updated occasionally, as new bills come in.

#### Source

Personal communication from the homeowner

inequality\_constraint *Graphics for constraints*

#### **Description**

These functions are intended to annotate contour plots with constraint regions. Inequality constraints are shaded where the constraint is NOT satisfied. Equality constraints are shaded in a small region near where the constraint is satisfied.

#### Usage

```
inequality_constraint(
 previous = NULL,
  tilde,
  domain,
  npts = 100,
  fill = "blue",alpha = 1)
equality_constraint(
 previous = NULL,
  tilde,
  domain,
  npts = 100.
  fill = "blue",
  alpha = 1)
```
#### <span id="page-23-0"></span>Arguments



#### Value

ggplot2 graphics layers

#### Examples

```
inequality\_constraint(x + y > 2 ~\sim~ y + x, \ domain(y=0:3, x=0:2))equality_constraint(x + y - 2 ~ y + x, domain(y=0:3, x=0:2), npts=200, alpha=.3, fill="red")
```


### Description

infer\_RHS turns a one-sided formula into a two-sided formula suitable for makeFun(). formals\_from\_expression creates a "formals" list for creating a function. The list will have arguments in the canonical order.

#### Usage

```
infer_RHS(ex)
```
formals\_from\_expr(ex, others = character(0))

#### Arguments



<span id="page-24-0"></span>inscribed\_circle *Create a data frame for a circle marking the curvature of a function.*

#### Description

Create a data frame for a circle marking the curvature of a function.

#### Usage

```
inscribed_circle(ftilde, x0 = 0)
```
#### Arguments



#### Value

inscribed\_circle() return a data frame with x and y components describing the inscribed circle located at the point  $(x_0, f(x_0))$ . Plot this with  $gf$ -path(y ~ x) curvature\_function() returns a function that gives the curvature of the specified function for any input \$x\$. It's much like D() in the way it is used.

Integrate *Integrate a function*

#### Description

Calculates the definite integral of a function. That is, the result of the integration will be a number. There can be no free parameters in the function being integrated. (If you want free parameters, use antiD().) Integrate() can handle integration over up to 3 variables.

#### Usage

```
Integrate(tilde, domain, ..., tol = 1e-05)
```
#### Arguments



#### <span id="page-25-0"></span>Details

For functions constructed as a spline interpolant, Integrate() can handle more segments than antiD(). It's reasonable to do up to 2000 segments with Integrate(), whereas antiD() handles only about 100.

#### Examples

```
Integrate(dnorm(x) \sim x, domain(x=-2:2))
Integrate(dnorm(x, sd=sigma) \sim x, domain(x=-2:2), sigma=2)
Integrate(sqrt(1- x^2) ~ x, domain(x=-1:1)) # area of semi-circle
```


#### Description

A formula interface to integration of an ODE with respect to "t"

#### Usage

```
integrateODE(...)
```
#### Arguments

... A dynamics object (see makeODE()) and/or arguments giving additional formulas for dynamics in other variables, assignments of parameters, assignments of initial conditions, the start and end times of the integration (through domain()), and the step size (through dt=).

#### Details

The equations must be in first-order form. Each dynamical equation uses a formula interface with the variable name given on the left-hand side of the formula, preceded by a d, so use  $dx$ <sup>-k\*</sup>x for exponential decay. All parameters (such as k) must be assigned numerical values in the argument list. All dynamical variables must be assigned initial conditions in the argument list. The returned value will be a list with one component named after each dynamical variable. The component will be a spline-generated function of t.

#### Value

a list with splined function of time for each dynamical variable

#### <span id="page-26-0"></span>is\_in\_domain 27

#### Examples

```
soln = integrateODE(dx~r*x*(1-x/k), k=10, r=.5, domain(t=0:20), x=1)
soln$x(10)
soln$x(30) # outside the time interval for integration
traj_plot(x(t)~t, soln, domain(t=0:10))
soln2 = integrate0DE(dx-y, dy--x, x=1, y=0, domain(t=0:10))traj_plot(y(t)~t, soln2)
SIR <- makeODE(dS~ -a*S*I, dI ~ a*S*I - b*I, a=0.0026, b=.5, S=762, I=1)
epi = integrateODE(SIR, domain(t=0:20))
traj_plot(I(t) ~ ~ t, epi)
```
is\_in\_domain *check whether a value is in a domain*

#### Description

A convenience function to see if a value is inside (or on the boundaries of) a domain or a set of domain segments. In the case of multiple segments, the check is whether val is within any of them.

#### Usage

is\_in\_domain(val, domain, ...)

#### Arguments



#### Examples

```
is_in_domain(1:10, domain(x=5.5:8.5), domain(x=1:2))
is_in_domain(mtcars, domain(mpg=15:20, wt=2:4.5), domain(mpg=25:28))
```


Iterate *Iterate a function on an initial condition*

#### Description

Iterates a function a specified number of times on an initial condition.

#### Usage

Iterate(f = NULL,  $A = NULL$ ,  $x0 = 0$ ,  $n = 10$ , fargs = list())

#### <span id="page-27-0"></span>Arguments



#### Details

The function f can take one or more arguments. The first of these should represent the *state* of the dynamical system, e.g. x, or x and y, etc. At the end of the argument list to f can come named parameters. The length of the initial condition  $x\theta$  must match the number of state arguments. Numerical values for parameters (if any) must be provided in the ... slot.

#### Value

A data frame with a column .i listing the iteration number and columns for each component of the initial condition. There will be n+1 rows, the first for the initial condition and the remaining for the n iterations.

#### Examples

```
Iterate(function(x, mu=3.5) mu*x*(1-x), x0=.232, n=10, list(mu=4)) # chaos
Iterate(function(x, y) c(x+y, x-y), x0 = c(1,1), n=10)Iterate(function(x, y) c(x+y, x), x0=c(1,0), n=10) # fibonacci
Iterate(A = cbind(rbind(1, 1), rbind(1, 0)), x0 = c(1, 0), n=5) # fibonacci described by a matrix
```
Kepler *Kepler's calculation of the position of Mars*

#### Description

Astronomer [Johannes Kepler](https://en.wikipedia.org/wiki/Johannes_Kepler) (1571-1630) famously measured the position of the planet Mars. His interpretation of this data led substantially to Newton's theory of universal gravity. This data frame gives Kepler's measurements as a function of time, together with a modern reconstruction of the "actual" position of Mars at the time.

#### Usage

data(Kepler)

#### <span id="page-28-0"></span> $M2014F$  29

#### Format

A data frame with 28 rows and these variables:

- time Interval in Earth days from 8:15am Greenwich time on 9 March 1584.
- kepler.radius The radius of the orbit in AU (astronomical units. 1 AU is 93 million miles)
- kepler.angle The "true anomoly" measured in radians
- actual.radius Modern calculation of the above
- actual.angle Modern calculation of the above

#### Details

The raw measurements (not included here) that Kepler used in his calculation were made by Tycho Brahe (1546-1601). Those raw measurements were of the angle of Mars with respect to Earth. Kepler estimated the orbital period of Mars to be 687 Earth days. (The current accepted value is 686.980 days.) Knowing the period, Kepler could find pairs of Earth days separated by multiples of the period. In each pair, the Earth would be in a different position, but Mars would be in the same position. Thus the distance of Mars from Earth could be estimated by triangulation.

The angle was not directly measured for each occasion. Instead, knowing the radius versus time Kepler was able to discern when Mars was at its greatest and closest distance to the Sun. The angle tells where Mars is along its orbit. An angle of 0 is the position when Mars is closest to the Sun. An angle of 3.14 is when Mars is farthest from the Sun.

#### Source

Drawn from from McLaughlin, Michael P. ( 1999 ) "A Tutorial on [Mathematical Modelling"](http://www.causascientia.org/math_stat/Tutorial.pdf) p. 21-23.

#### References

See https://faculty.uca.edu/saustin/3110/mars.pdf for a useful description of the estimation process.

M2014F *Mortality versus age for females in the US in 2014*

#### Description

These data are from the US Social Security Administration.

#### Usage

M2014F

#### Format

An object of class tbl\_df (inherits from tbl, data.frame) with 120 rows and 5 columns.

#### <span id="page-29-0"></span>Details

- pmort the probability of a person of the given age dying in 2014
- nliving traces a hypothetical population of 100,000 people through the mortality at each age.
- died for the hypothetical population the number of people at that age who survived to the age and then died at that age. Source: [https://www.ssa.gov/oact/HistEst/PerLifeTables/](https://www.ssa.gov/oact/HistEst/PerLifeTables/2017/PerLifeTables2017.html) [2017/PerLifeTables2017.html](https://www.ssa.gov/oact/HistEst/PerLifeTables/2017/PerLifeTables2017.html)



#### Description

An ODE object consists of some dynamics, initial conditions, parameter values, time domain, etc.

#### Usage

makeODE(...)

#### Arguments

... The components of an ODE and/or a set of other ODE objects. By default, the time step is set to  $dt = 0.01$ . Change it with  $dt =$  value.

#### Value

a list containing various functions and values specifying the ODE.

#### Examples

```
SIR <- makeODE(dS~ -a*S*I, dI ~ a*S*I - b*I, a=0.0026, b=.5, S=762, I=1)
soln <- integrateODE(SIR, domain(t=0:20))
```
<span id="page-29-1"></span>

numD *Numerical Derivatives*

#### Description

Constructs the numerical derivatives of mathematical expressions

#### Usage

 $numD(tilde, ..., h = NULL)$ 

#### <span id="page-30-0"></span> $\mu$ numD 31

#### Arguments



#### Details

Uses a simple finite-difference scheme to evaluate the derivative. The function created will not contain a formula for the derivative. Instead, the original function is stored at the time the derivative is constructed and that original function is re-evaluated at the finitely-spaced points of an interval. If you redefine the original function, that won't affect any derivatives that were already defined from it. Numerical derivatives, particularly high-order ones, are unstable. The finite-difference parameter .h is set, by default, to give reasonable results for first- and second-order derivatives. It's tweaked a bit so that taking a second derivative by differentiating a first derivative will give reasonably accurate results. But, if taking a second derivative, much better to do it in one step to preserve numerical accuracy.

#### Value

a function implementing the derivative as a finite-difference approximation. This has a second argument, .h, that allow the finite-difference to be set when evaluating the function. The default values are set for reasonable numerical precision.

#### Note

WARNING: In the expressions, do not use variable names beginning with a dot, particularly . f or .h

#### Author(s)

Daniel Kaplan (<kaplan@macalester.edu>)

#### Examples

```
g = numD( a*x^2 + x*y - x, a=1)g(x=2,y=10)
gg = numD( a*x^2 + x*y - x&x, a=1)gg(x=2,y=10)
ggg = numD( a*x^2 + x*y - x&y, a=1)ggg(x=2,y=10)
h = numD( g(x=x,y=y, a=a) ~ x y, a=1)h(x=2, y=10)f = numD(jsin(x)~x)# slice_plot( f(3,.h=hlim)~h, bounds(h=.00000001 :000001)) %>% gf_hline(yintercept = cos(3))
```
<span id="page-31-0"></span>

#### Description

These appear in the "Optimization & constraint" chapter.

#### Usage

PE\_fun1(x1, y1) PE\_fun2(x1, y1, x2, y2, x3, y3)

#### Arguments

x1, y1, x2, y2, x3, y3

coordinates of the masses.

Picket *Creates a "picket fence" of points for illustrating numerical integration*

#### Description

Creates a "picket fence" of points for illustrating numerical integration

#### Usage

```
Picket(
  domain,
  h = 0.1,
  method = c("left", "right", "center", "trapezoid", "gauss")
\mathcal{L}
```
#### Arguments



<span id="page-32-0"></span>

#### Description

NASA data on planets

#### Usage

data(Planets)

#### Format

A data frame with 10 rows and these variables:

- planet name of the planet
- mass in  $10^{\texttt{A}}24$  kg
- diameter in km
- density in kg/m^3
- gravity in m/s^2
- escape\_velocity in km/s
- rotational\_period in hours
- day\_length in hours
- distance\_from\_sun in 10^6 km
- perihelion in 10^6 km
- aphelion in 10^6 km
- orbital\_period in days
- orbital\_velocity in km/s
- orbital\_inclination in degrees
- orbital\_eccentricity
- obliquity\_to\_orbit in degrees
- mean\_temperature in C
- surface\_pressure in bars
- n\_moons a count
- ring\_system does the planet have a ring system
- global\_magnetic\_field does the planet have a magnetic field

#### Details

Two of the rows, MOON and PLUTO, are not considered planets. The orbital parameters of the MOON are with respect to EARTH. Others are with respect to the Sun. Negative rotation period denotes a rotation opposite that of most of the other planets.

<span id="page-33-0"></span>https://nssdc.gsfc.nasa.gov/planetary/factsheet/

Planet\_solar *Solar irradiance of the planets*

#### Description

Brightness of the sun at the various planets

#### Usage

data(Planet\_solar)

#### Format

A data frame with 9 rows

- planet Name of the planet (or planetoid)
- distance distance from the sun in millions of km
- irradiance solar brightness (in what units?)

<span id="page-33-1"></span>plotFun *Plot functions of one and two variables using lattice system*

#### Description

This is defined in the mosaic package: See [plotFun](#page-33-1)

qspliner *Create a quadratic spline (inefficiently)*

#### Description

A handmade function to construct quadratic splines.

#### Usage

qspliner(tilde, data, free = 0)

#### <span id="page-34-0"></span>**Arguments**



#### Details

Unless you have a good reason otherwise, you should be using spliner(), which generates cubic splines, rather than qspliner(). qspliner() is intended only for demonstration purposes.

#### Value

a function suitable for, for instance, graphing or optimizing

#### Examples

```
Pts \leftarrow tibble(x = seq(-4,4, by=.7), y = dnorm(x))
f \leftarrow qspliner(y \sim x, data = Pts)
slice_plot(dnorm(x) ~ x, domain(x=-4:4)) %>%
  slice_plot(f(x) \sim x, color= "blue") %>%
  gf\_point(y \sim x, data = Pts, color = "orange", size=4, alpha=0.3) %>%
  gf<sub>-</sub>lims(y= c(NA,.5))
```
<span id="page-34-1"></span>rfun *Generate a "natural looking" function of one or multiple variables*

#### Description

This is defined in the mosaic package: See [rfun](#page-34-1).

RI\_tide *Tide levels from the US NOAA*

#### Description

- RI\_tide: For Providence, RI, Minute-by-minute tide data during April 1-5 2010.
- Anchorage\_tide: For Anchorage, AK, Every six-minute data during 2018

#### Usage

RI\_tide

#### Format

An object of class spec\_tbl\_df (inherits from tbl\_df, tbl, data.frame) with 6426 rows and 3 columns.

#### Details

Variables:

- level meters
- date\_time GMT
- hour time in hours after start of April 2010.

Lat/Long: 41.8071N,-71.4012W Request: https://tidesandcurrents.noaa.gov/api/datagetter?begin\_date=20200 Other product documentation: https://tidesandcurrents.noaa.gov/api/ Google Map: https://www.google.com/maps/place/Prov 71.4012,14z/data=!4m5!3m4!1s0x89e444e0437e735d:0x69df7c4d48b3b627!8m2!3d41.8071!4d-71.4012

Robot\_stations *Waypoints on the path of a fictitious robot*

#### Description

This is used in *MOSAIC Calculus* examples about splining.

#### Usage

Robot\_stations

#### Format

A data frame with 16 cases, each of which is an x-y location at the indicated time.

- x,y X-Y location in mm
- t time in seconds

<span id="page-35-0"></span>

<span id="page-36-0"></span>

#### Description

A longitudinal record of running times. There are 5977 individual runners included here, each with a unique id. Each runner ran the 10-mile race (See mosaicData::TenMileRace) on multiple occasions.

#### Usage

Runners

#### Format

Data frame with 24,334 rows

- year the occasion on which the runner ran the Cherry Blossom Ten Miler
- age the runners age at the time of that occasion
- gun the time (in minutes) from the gun to crossing the finish line.
- net time between crossing the start line and crossing the finish line. Sometimes this is missing. It's typically smaller than gun because it takes time to reach the starting line.
- sex as recorded by the race organizers.
- previous How many previous occasions did this person run the race.
- nruns The total number of runs recorded for this person (including future runs)
- start\_position A qualitative indication of how close to the front of the pack the person was positioned at the time the gun went off.

#### Author(s)

Daniel Kaplan

#### Source

Scraped from the Cherry Blossom web site by patient, unremunerated toil of the author.

<span id="page-37-0"></span>simpleYacasIntegrate *Interface to integration using Ryacas*

### Description

Interface to integration using Ryacas

#### Usage

```
simpleYacasIntegrate(tilde, ...)
```
#### Arguments





### Description

Turn a 1-line function into an inline formula

#### Usage

```
simplify_fun(fun)
```
inline\_expr(ex, old, new, env)

replace\_arg\_in\_expr(ex, old, new)

#### Arguments



<span id="page-38-0"></span>

#### Description

In a slice plot, there is one independent variable. The graph shows the output of the function versus the independent variable. It's called a slice plot to distinguish it from a contour plot, in which the graph has one axis for each independent variable and the output of the function is shown by color and labels.

#### Usage

```
slice_plot(
  ...,
 npts = 101,
 color = "black",
 alpha = 1,
 label\_text = "",label_x = 1,
  label_vjust = "top",
  label_color = color,
  label_alpha = alpha,
  singularities = numeric(0)
)
```
#### Arguments



#### Value

ggplot2 layers

#### Examples

```
## Not run:
slice_plot(sin(x) \sim x, domain(x = range(-5, 15)))
f <- makeFun(sin(2*pi*t/P) \sim t)
slice_plot(f(t, P=20) ~ t, domain(t = -5:10), label_text = "Period 20", label_x=0.9)
slice_plot(x^2 ~ x) # Error: no domain specified
slice_plot(cos(x) ~ x, domain(x[-10:10]) # domain will be -10 < x < 10
# see domain
```
## End(Not run)

<span id="page-39-1"></span>smoother *Create a smoothing function approximating a cloud of points*

#### Description

This is defined in the mosaic package: See [smoother](#page-39-1).

SSA\_2007 *US Mortality table from 2007*

#### **Description**

Mortality table from the US Social Security Adminstration issued in 2007.

#### Usage

SSA\_2007

#### Format

A data frame with 240 rows. Each row corresponds to one age year from 0 to 120.

- Age: The age year. 0 corresponds to birth through one year of age.
- Sex: The sex for which the row is relevant.
- Mortality: The fraction of people of that age who died in 2007.
- LifeExpectancy: The calculated "life expectancy" at that age for that sex.

#### Details

Life expectancy is a measure constructed from a simulation. Start with 100,000 people at birth. Use the mortality at each age to follow the living through successive ages. The "life expectancy at birth" (age 0) is the average age at death of those 100,000 people. For the life expectancy in year n, consider only that fraction of the original 100,000 who survived to year n. The life expectancy will be the average time until death for those survivors.

<span id="page-39-0"></span>

#### <span id="page-40-0"></span>Description

- streamlines() draws raindrop-shaped paths at a randomized grid of points that follow trajectories
- flow\_field() draws arrows showing the flow at a grid of points
- trajectory\_euler() compute an Euler solution. ()

#### Usage

```
streamlines(..., npts = 8, dt = 0.01, nsteps = 10, color = "black", alpha = 1)
flow_field(..., npts = 8, scale = 0.8, color = "black", alpha = 1)
trajectory_euler(..., dt = 0.01, nsteps = 4, full = TRUE, every = 1)
```
#### Arguments



#### Details

The dynamical functions themselves will be formulas like  $dx \sim a \times x \times y$  and  $dy \sim y/x$ . Initial conditions will be arguments of the form  $x=3$  and  $y=4$ . If there are parameters in the dynamical functions, you should also add the parameter values, for instance a=2.

For flow\_field() and streamlines() you do not need to specify initial conditions. The grid will be set by the domain argument.

The graphics functions are all arranged to accept, if given, a ggplot object piped in. The new graphics layer will be drawn on top of that. If there is no ggplot object piped in, then the graphics will be made as a first layer, which can optionally piped into other ggformula functions or + into ggplot layers.

#### Examples

```
streamlines(dx \sim x+y, dy\sim x-y, domain(x=0:6, y=0:3))
flow_field(dx \sim x+y, dy\sim x-y, domain(x=0:6, y=0:3))
Dyn <- makeODE(dx \sim 0.06*x, dy \sim-x - y, domain(t=0:20), dt=0.1,x=3, y=4)
streamlines(Dyn, domain(x=-5:5, y=-5:5), npts=15)
flow_field(Dyn, domain(x=-6:6, y=-10:10))
trajectory_euler(dx \sim -y, dy \sim .5*x, x=3, y=3)
rabbits <- drabbit ~ 0.2*rabbit - 0.01*rabbit*fox
foxes \leq dfox \sim -.2*fox + 0.0005*rabbit*fox
trajectory_euler(rabbits, foxes, rabbit=100, fox=3, dt=0.1, nsteps=500, every=10)
```
surface\_plot *Make an interactive plotly plot of a function of two variables*

#### Description

An interactive plot lets you interrogate the plot to get numerical values at each point. When type = "surface", the plot can be rotated to see the "shape" of the function from various perspectives. Using the interactive controls, you can save the plot as a PNG file. But it's not possible to overlay plots the way you can with contourPlot().

#### Usage

```
surface_plot(
  formula,
  domain = c(-5, 5),
 npts = 50,
  type = c("both", "surface", "contour", "heatmap")
\lambdainteractive_plot(
  formula,
 domain = c(-5, 5),
 npts = 50,
  type = c("both", "surface", "contour", "heatmap")
)
```
surface\_with\_contours(formula, domain =  $c(-5, 5)$ , npts = 50)

#### Arguments



<span id="page-41-0"></span>

#### <span id="page-42-0"></span>symbolicD 43

#### Examples

```
## Not run:
interactive_plot(
    sin(fred*ginger) \sim fred + ginger,
    domain(fred=range(0,pi),
           ginger = range(0, pi),
    type = "both")
```
## End(Not run)

symbolicD *Symbolic Derivatives*

#### Description

Constructs symbolic derivatives of some mathematical expressions

#### Usage

symbolicD(tilde, ..., .order)

#### Arguments



#### Details

Uses the Derivs package for constructing the derivative The .order argument is just for convenience when programming high-order derivatives, e.g. the 5th derivative w.r.t. one variable.

When re-assigning default values for arguments in a function being called, as in  $D(\text{dnorm}(x,$ mean=3)  $\sim$  x), you will get a numerical derivative even when the analytic form is known. To avoid this (when possible) use  $D(\text{dnorm}(x) \sim x, \text{mean=3})$ 

#### Value

a function implementing the derivative

#### Author(s)

Daniel Kaplan (<kaplan@macalester.edu>)

#### See Also

[D](#page-12-1), [numD](#page-29-1), [antiD](#page-12-2), [plotFun](#page-33-1)

#### Examples

```
symbolicD(a*x^2 \sim x)
symbolicD( a*x^2 ~ x&x)
symbolicD( a*sin(x)~x, .order=4)
symbolicD(a*x^2*y+b*y \sim x, a=10, b=100)
symbolicD( dnorm(x, mn, sd) \sim x, mn=3, sd=2)
```
traj\_plot *Plots a trajectory*

#### Description

This function handles trajectories can stem from either of two sources:

- 1. A parametric description of a curve, such as  $sin(t) \sim cos(t)$ , along with a domain in t.
- 2. The solution to an ordinary differential equation as produce by integrateODE()

#### Usage

traj\_plot $(..., npts = 500, nt = 5)$ 

#### Arguments



#### Details

The tilde expression is the critical part

#### Examples

```
traj_plot(2*x + 3 \sim sin(x), domain(x=0:10))
PPdyn <- makeODE(dR ~ 0.3*R - 0.03*R*F, dF ~ -0.3*F + 0.0003*R*F)
Soln <- integrateODE(PPdyn, domain(t=0:20), R=1200, F=8)
traj_plot(R(t) \sim F(t), Soln, nt=10)
traj_plot(R(t)*F(t) ~ t, Soln, nt=0)
```
<span id="page-43-0"></span>

<span id="page-44-0"></span>

#### Description

Takes a trajectory with three state variables as produced by integrateODE() and plots out in a 3-dimensional perspective plot, which can be rotated.

#### Usage

 $traj_plot_3D(x, y, z, soln, domain = NULL, npts = 1000)$ 

#### Arguments



#### Examples

```
Lorenz <- makeODE(dx ~ sigma*(y-x), dy ~(x*(rho-z) - y), dz ~ (x*y - beta*z),
                  rho=28, sigma=10, beta = 8/3)
T1 <- integrateODE(Lorenz, domain(t=0:50), x=-5, y=-7, z=19.4)
traj_plot_3D(x, y, z, T1, npts=5000)
```
UK\_GDP *Gross Domestic Product of the United Kingdom over a millenium*

#### Description

An estimate of the real GDP of the United Kingdom over about 7 centuries. "Real" means adjusted for inflation. Take such estimates with a large amount of salt, since pre-industrial era data is very limited and because the components of GDP change substantially over a few decades, let alone a few centuries. Also, the name UK\_GDP is anachronistic, since for most of the record is before the UK was created as a political entity.

#### Usage

data(UK\_GDP)

#### Format

A data frame with 747 rows

- year Which year the estimate is for.
- GDP *per capita* Gross domestic product in GBP (pounds)

unbound *Identifying unbound inputs to a function*

#### Description

unbound() finds if there are any unbound parameters in a function. This can be useful for checking before handing a function over to a numerical routine. bind\_params() lets you add parameter bindings or override existing ones.

#### Usage

unbound(f)

bind\_params(f, ...)

#### Arguments



US\_income *Income distribution data from the US in 2009*

#### Description

Data such as these are used to study income inequality. One measure of this is the "Gini coefficient," which can be estimated from a Lorenz function fitted to such data.

#### Usage

US\_income

#### Format

An object of class tbl\_df (inherits from tbl, data.frame) with 6 rows and 2 columns.

<span id="page-45-0"></span>

#### <span id="page-46-0"></span>vectors and the contract of th

#### Source

La Haye and Zizler (2021) "The Lorenz Curve in the Classroom", *The American Statistician*, 75(2):217-225]

Variables:

- income The fraction of total household income that goes to the corresponding poorest fraction of the population.
- pop The fraction of the population whose aggregated income is reported as the income value.

vectors *Utilities for vector calculations*

#### Description

%dot%, %onto%, and %perp% are infix operators. The left-hand argument is a vector.

#### Usage



#### Arguments



#### Details

Convenience functions for basic operations relating to vector projection. These use the *MOSAIC Calc* conventions that require vectors to be one column matrices.

#### Value

either a number (for %dot%) or a vector

<span id="page-47-0"></span>48 Vowel\_ee

vector\_arg *convert a function with separate arguments into one with a single vector argument For use with optim.*

#### Description

convert a function with separate arguments into one with a single vector argument For use with optim.

#### Usage

vector\_arg(f)

#### Arguments

f a function with multiple arguments

Vowel\_ee *Recordings of vowel sounds*

#### Description

"Oh" refers to "o" as in "stone." "Ee" to "eel".

#### Usage

Vowel\_ee

Vowel\_oh

Oh\_sound

Ee\_sound

#### Format

Data frames with two columns

- y: The instantaneous amplitude of the vowel sound.
- t: Time (seconds)

An object of class tbl\_df (inherits from tbl, data.frame) with 780 rows and 2 columns. An object of class tbl\_df (inherits from tbl, data.frame) with 7192 rows and 2 columns. An object of class tbl\_df (inherits from tbl, data.frame) with 10194 rows and 2 columns.

#### <span id="page-48-0"></span>Zeros 49

#### Details

- Oh\_sound and Ee\_sound contain the complete utterances.
- Vowel\_ee and Vowel\_oh are snippets of about 0.04 seconds duration.



Zeros *Finds zeros of a function within a specified domain*

#### Description

Finds zeros of a function within a specified domain

#### Usage

Zeros(tilde, domain =  $NULL, ..., nsegs = 131)$ 

#### Arguments



#### Value

A data frame with two columns. The first has the name of the input in the tilde expression, and gives the values for that input at which the function is approximately zero. The second column, .output. gives the actual value of the function at the inputs in the first column.

#### Examples

Zeros( $a*x + b \sim x$ ,  $a=1$ ,  $b=2$ )

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