Package ‘CircNNTSRmult’

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Type Package

Title Multivariate Circular Data using MNNTS Models

Version 0.1.0

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Description A collection of utilities for the statistical analysis of multivariate circular data using distributions based on Multivariate Nonnegative Trigonometric Sums (MNNTS). The package includes functions for calculation of densities and distributions, for the estimation of parameters, and more.

Depends R (>= 3.5.0), stats, psychTools, CircNNTSR

License GPL (>= 2)

LazyLoad yes

NeedsCompilation no

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`CircNNTSRmult-package` *Multivariate Circular Data using MNNTS Models*

Description

A collection of utilities for the statistical analysis of multivariate circular data using distributions based on Multivariate Nonnegative Trigonometric Sums (MNNTS). The package includes functions for calculation of densities and distributions, for the estimation of parameters, and more.

Details

Package:	CircNNTSRmult
Type:	Package
Version:	0.1.0
Date:	2023-09-09
License:	GLP (>=2)
Depends:	R (>= 3.5.0), stats, psychTools, CircNNTSR
LazyLoad:	yes
NeedsCompilation:	no

The MNNTS (multivariate NNTS) density on a d-dimensional ($d > 2$) hypertorus by Fernandez-Duran and Gregorio-Dominguez (2014) (see also Fernandez-Duran and Gregorio-Dominguez, 2016) for a vector of angles, $\underline{\Theta} = (\Theta_1, \Theta_2, \ldots, \Theta_d)^\top$, is defined as

$$f_{\underline{\Theta}}(\underline{\theta}) = \frac{1}{(2\pi)^d} \underline{c}_{12\cdots d}^H \underline{e}\underline{e}^H \underline{c}_{12\cdots d}$$

$$= \frac{1}{(2\pi)^d} \sum_{k_1=0}^{M_1} \sum_{k_2=0}^{M_2} \cdots \sum_{k_d=0}^{M_d} \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} \cdots \sum_{m_d=0}^{M_d} c_{k_1 k_2 \cdots k_d} \bar{c}_{k_1 k_2 \cdots k_d} e^{\sum_{r=1}^{d} i(k_r - m_r)\theta_r}$$

where $\underline{c}_{12\cdots d}$ is a d-dimensional parameter vector of complex numbers of dimension $2\prod_{r=1}^{d}(M_r + 1) - 1$ with subindexes given for all the combinations (Kronecker products) of the d vectors $\underline{M}_r = (0, 1, \ldots, M_r)^\top$ for $r = 1, 2, \ldots, d$ where M_r is the number of terms of the sum in the equation for the r-th component of the vector $\underline{\Theta}$. The vector $\underline{c}_{12\cdots d}$ must satisfy $\underline{c}_{12\cdots d}^H \underline{c}_{12\cdots d} = ||\underline{c}_{12\cdots d}||^2 = \sum_{k_1=0}^{M_1} \sum_{k_2=0}^{M_2} \cdots \sum_{k_d=0}^{M_d} ||c_{k_1 k_2 \cdots k_d}||^2 = 1$. For identifiabily, $c_{00\cdots 0}$ is a nonnegative real number. The vector $\underline{c}_{12\cdots d}^H$ is the Hermitian (conjugate and transpose) of vector $\underline{c}_{12\cdots d}$. The MNNTS family has many desirable properties, the marginal and conditional densities of any order of an MNNTS density are also MNNTS densities and, independence among the elements of the vector $\underline{\Theta}$ is translated into a Kronecker product decomposition in the parameter vector $\underline{c}_{12\cdots d}$. For example, in the trivariate case $\underline{\Theta} = (\Theta_1, \Theta_2, \Theta_3)^\top$, if Θ_1, Θ_2 and Θ_3 are joint independent then, $\underline{c}_{123} = \underline{c}_1 \bigotimes \underline{c}_2 \bigotimes \underline{c}_3$ where $\underline{c}_1$, $\underline{c}_2$ and $\underline{c}_3$ are the parameter vectors of the NNTS marginal densities of Θ_1, Θ_2 and Θ_3, respectively. Similarly, if Θ_1 is groupwise independent of $(\Theta_2, \Theta_3)^\top$ then, $\underline{c}_{123} = \underline{c}_1 \bigotimes \underline{c}_{23}$ where $\underline{c}_{23}$ is the parameter vector of the bivariate MNNTS density of $(\Theta_2, \Theta_3)^\top$. These results apply to higher dimensions.

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

Maintainer: Maria Mercedes Gregorio Dominguez <mercedes@itam.mx>

References

Fernandez-Duran, J. J. and Gregorio-Dominguez M. M. (2014) Modeling angles in proteins and circular genomes using multivariate angular distributions based on nonnegative trigonometric sums. *Statistical Applications in Genetics and Molecular Biology*, 13(1), 1-18.

Fernandez-Duran, J. J. and Gregorio-Dominguez, M. M. (2016). CircNNTSR: an R package for the statistical analysis of circular, multivariate circular, and spherical data using nonnegative trigonometric sums. *Journal of Statistical Software*, 70, 1–19.

Fernandez-Duran, J. J. and Gregorio-Dominguez, M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data, *arXiv preprint arXiv:2301.03643v2*

`EURUSDGBPBTCtimesminmax` *Minimun and Maximun daily exchange rates*

Description

Minimun and maximun daily bid and ask exchange rates from Euro-US dollar, GB pound-US dollar, Bitcoin-US dollar from March 22, 2019 to March 22, 2023

Usage

```
data("EURUSDGBPBTCtimesminmax")
```

Format

A data frame with 1048 observations on the following 14 variables.

`id` Observation number

`day1` Date in format day/month/year

`EURUSDAskMax` Daily maximum of ask Euro-US dollar exchange rate

`EURUSDAskMin` Daily minimum of ask Euro-US dollar exchange rate

`EURUSDBidMax` Daily maximum of bid Euro-US dollar exchange rate

`EURUSDBidMin` Daily minimum of bid Euro-US dollar exchange rate

`GBPUSDAskMax` Daily maximum of ask GB pound-US dollar exchange rate

`GBPUSDAskMin` Daily minimum of ask GB pound-US dollar exchange rate

`GBPUSDBidMax` Daily maximum of bid GB pound-US dollar exchange rate

`GBPUSDBidMin` Daily minimum of bid GB pound-US dollar exchange rate

`BTCUSDAskMax` Daily maximum of ask Bitcoin-US dollar exchange rate

`BTCUSDAskMin` Daily minimum of ask Bitcoin-US dollar exchange rate

`BTCUSDBidMax` Daily maximum of bid Bitcoin-US dollar exchange rate

`BTCUSDBidMin` Daily minimum of bid Bitcoin-US dollar exchange rate

Source

Dukascopy publicly available tick-by-tick data

`mnntestimationresultantvector`
c Parameter Vector Estimate

Description

Computes the c parameter vector estimate based on the mean resultant vector of the vectors of observed trigonometric moments

Usage

```
mnntestimationresultantvector(data,M=0,R=1)
```

Arguments

`data`	Data frame with the observed vectors of angles. The number of columns must be equal to R
`M`	Vector of M parameters. A nonnegative integer number for each of the R components of the vector
`R`	Number of dimensions

Value

`cestimates`	A matrix with the index and values of the c parameters estimates of the MNNTS density

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data, arXiv preprint arXiv:2301.03643v2

Examples

```
# A bivariate dataset

Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest*(pi/180)
estmeanresultant<-mnntestimationresultantvector(data,M=Mbiv,R=Rbiv)
```

```
estmeanresultant

# A trivariate dataset

Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
data<-WindDirectionsTrivariate
estmeanresultant<-mnntestimationresultantvector(data,M=Mtriv,R=Rtriv)
estmeanresultant
```

`mnntscharacteristicfunction`
Characteristic Function of an MNNTS Density

Description

Computes the characteristic function from the c parameters of an MNNTS density

Usage

```
mnntscharacteristicfunction(cestimatesarray=as.data.frame(matrix(c(0,1/(2*pi)),
nrow=1,ncol=2)),M=0,R=1)
```

Arguments

`cestimatesarray`
output from mnntsmanifoldnewtonestimation function

`M` Vector of M parameters. A nonnegative integer number for each of the R components of the vector

`R` Number of dimensions

Value

A data frame (matrix) with the support and values of the characteristic function of the MNNTS density

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data, arXiv preprint arXiv:2301.03643v2

Examples

```
# A characteristic function from a bivariate MNNTS density

set.seed(200)
Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest*(pi/180)
est<-mnntsmanifoldnewtonestimation(data,Mbiv,Rbiv,50)
est
charfunbiv23<-mnntscharacteristicfunction(cestimatesarray=est$cestimates,M=Mbiv,R=Rbiv)
charfunbiv23

# A characteristic function from a trivariate MNNTS density

set.seed(200)
Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
data<-WindDirectionsTrivariate
est<-mnntsmanifoldnewtonestimation(data,Mtriv,Rtriv,50)
est
charfuntriv233<-mnntscharacteristicfunction(cestimatesarray=est$cestimates,M=Mtriv,R=Rtriv)
charfuntriv233
```

---

mnntsconditional *Conditional MNNTS density*

Description

Computes the c parameters of a conditional MNNTS density at a particular value of the conditioning random vector

Usage

```
mnntsconditional(cpars=as.data.frame(matrix(c(0,0,1/(2*pi)),nrow=1,ncol=3)),
M=c(0,0),R=2,cond=1,cond.values=0)
```

Arguments

cpars	Matrix of parameters of an MNNTS density with the first R columns containing the index of the c parameter and the R+1 containing the complex parameter
M	Vector of M parameters. A nonnegative integer number for each of the R components of the vector
R	Number of dimensions
cond	A subset of 1:R indicating the elements of the vector of variables to conditioning on
cond.values	A vector of fixed values of the conditional elements of the random vector at which to conditioning on

Value

param A matrix with the index and values of the c parameters for the MNNTS conditional density

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data, arXiv preprint arXiv:2301.03643v2

Examples

```
# A univariate conditional from a bivariate joint

set.seed(200)
Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest*(pi/180)
est<-mnntsmanifoldnewtonestimation(data,Mbiv,Rbiv,100)
est
cpars2cond1<-mnntsconditional(cpars=est$cestimates,M=Mbiv,R=Rbiv,cond=1,cond.values=c(pi/2))
cpars2cond1
nntsplot(cpars2cond1$cpar.cond,M=Mbiv[2])

# A bivariate conditional from a trivariate joint

set.seed(200)
Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
data<-WindDirectionsTrivariate
est<-mnntsmanifoldnewtonestimation(data,Mtriv,Rtriv,100)
est
cpars23cond1<-mnntsconditional(cpars=est$cestimates,M=Mtriv,R=Rtriv,cond=1,cond.values=pi/4)
cpars23cond1
mnntsplot(cpars23cond1,M=Mtriv[c(2,3)])
mnntsplotwithmarginals(cpars23cond1,M=Mtriv[c(2,3)])
```

---

mnntsgofdesignmatrix *Design Matrix of the MNNTS Goodness of Fit Test*

Description

Computes the design matrix of the auxiliary regression for the goodness of fit test of an MNNTS density based on the estimated characteristic function

Usage

```
mnntsgofdesignmatrix(data,charfunarray,R=1)
```

Arguments

`data`	Matrix of angles in radians (with R columns)
`charfunarray`	A data frame (matrix) with the support and values of the characteristic function of the MNNTS density obtained by using the function mnntscharacteristic function with vector of parameters M of dimension R
`R`	Number of dimensions

Value

A matrix that is the design matrix to run the auxiliary regression for the goodness of fit test

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data, arXiv preprint arXiv:2301.03643v2

Fan, Y. (1997). Goodness-of-fit tests for a multivariate distribution by the empirical characteristic function. Journal of Multivariate Analysis, 62, 36-63.

Examples

```
# A characteristic function from a bivariate MNNTS density

set.seed(200)
Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest*(pi/180)
est<-mnntsmanifoldnewtonestimation(data,Mbiv,Rbiv,70)
est
charfunbiv23<-mnntscharacteristicfunction(cestimatesarray=est$cestimates,M=Mbiv,R=Rbiv)
charfunbiv23
designmatrix23<-mnntsgofdesignmatrix(data,charfunbiv23,R=2)
designmatrix23

# A characteristic function from a trivariate MNNTS density

set.seed(200)
Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
```

```
data<-WindDirectionsTrivariate
est<-mnntsmanifoldnewtonestimation(data,Mtriv,Rtriv,40)
est
charfuntriv233<-mnntscharacteristicfunction(cestimatesarray=est$cestimates,M=Mtriv,R=Rtriv)
charfuntriv233
designmatrix233<-mnntsgofdesignmatrix(data,charfuntriv233,R=3)
designmatrix233
```

`mnntsgofstatistics` *Statistics of the MNNTS Goodness of Fit Test*

Description

Computes the statistics of the goodness of fit test of an MNNTS density based on the estimated characteristic function

Usage

```
mnntsgofstatistics(data,charfunarray,R=1)
```

Arguments

`data`	Matrix of angles in radians (with R columns)
`charfunarray`	A data frame (matrix) with the support and values of the characteristic function of the MNNTS density obtained by using the function mnntscharacteristicfunction with vector of parameters M of dimension R
`R`	Number of dimensions

Value

`gofstat`	The value of the goodness of fit statistic
`gofstatnormal`	The value of the normal approximation o fthe goodnes of fit statistic

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data,arXiv preprint arXiv:2301.03643v2

Fan, Y. (1997). Goodness-of-fit tests for a multivariate distribution by the empirical characteristic function. Journal of Multivariate Analysis, 62, 36-63.

Examples

```
# A characteristic function from a bivariate MNNTS density

set.seed(200)
Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest*(pi/180)
est<-mnntsmanifoldnewtonestimation(data,Mbiv,Rbiv,70)
est
charfunbiv23<-mnntscharacteristicfunction(cestimatesarray=est$cestimates,M=Mbiv,R=Rbiv)
charfunbiv23
gofstats23<-mnntsgofstatistics(data,charfunbiv23,R=2)
gofstats23

# A characteristic function from a trivariate MNNTS density

set.seed(200)
Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
data<-WindDirectionsTrivariate
est<-mnntsmanifoldnewtonestimation(data,Mtriv,Rtriv,50)
est
charfuntriv233<-mnntscharacteristicfunction(cestimatesarray=est$cestimates,M=Mtriv,R=Rtriv)
charfuntriv233
gofstats233<-mnntsgofstatistics(data,charfuntriv233,R=3)
gofstats233
```

---

mnntsmarginalgeneral *Mixing Probabilities of the Elements of the Mixture*

Description

Computes the mixing probabilities (eigenvalues) and parameter c vectors (eigenvectors) of the elements of the mixture defining a general MNNTS marginal of any dimension from an MNNTS density

Usage

```
mnntsmarginalgeneral(cpars=as.data.frame(matrix(c(0,0,1/(2*pi)),nrow=1,ncol=3)),
M=c(0,0),R=2,marginal=1)
```

Arguments

cpars	Matrix of parameters of an MNNTS density with the first R columns containing the index of the c parameter and the R+1 containing the complex parameter
M	Vector of M parameters. A nonnegative integer number for each of the R components of the vector

R	Number of dimensions
marginal	A subset of 1:R indicating the elements of the random vector in the marginal

Value

index	Matrix of the index of the marginal MNNTS density
eigenvectors	Matrix of the c parameter vectors of each element of the mixture. Each column is a parameter vector
eigenvalues	The vector of mixing probabilities

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data, arXiv preprint arXiv:2301.03643v2

Examples

```
# A univariate marginal from a bivariate joint

set.seed(200)
Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest*(pi/180)
est<-mnntsmanifoldnewtonestimation(data,Mbiv,Rbiv,100)
est
cparsmarginal1<-mnntsmarginalgeneral(cpars=est$cestimates,M=Mbiv,R=Rbiv,marginal=1)
cparsmarginal1

# A bivariate marginal from a trivariate joint

set.seed(200)
Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
data<-WindDirectionsTrivariate
est<-mnntsmanifoldnewtonestimation(data,Mtriv,Rtriv,100)
est
cparsmarginal12<-mnntsmarginalgeneral(cpars=est$cestimates,M=Mtriv,R=Rtriv,marginal=c(1,2))
cparsmarginal12
```

mnntsmarginalgeneraldimension

Marginal Density Function at a Vector of Fixed Values

Description

Computes the value of the marginal density function at a set of vector of angles

Usage

```
mnntsmarginalgeneraldimension(cpars=as.data.frame(matrix(c(0,0,1/(2*pi)),nrow=1,
ncol=3)),M=c(0,0),R=2,marginal=1,theta=matrix(0,nrow=1,ncol=1))
```

Arguments

cpars	Matrix of parameters of an MNNTS density with the first R columns containing the index of the c parameter and the R+1 containing the complex parameter
M	Vector of M parameters. A nonnegative integer number for each of the R components of the vector
R	Number of dimensions
marginal	A subset of 1:R indicating the elements of the vector of variables in the marginal
theta	A vector of fixed values of the marginal elements of the random vector at which to obtain the value of the marginal density

Value

A scalar with the value of the marginal density at the specified value of the marginal vector.

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions f

Examples

```
# A univariate marginal from a bivariate joint

set.seed(200)
Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest
est<-mnntsmanifoldnewtonestimation(data,Mbiv,Rbiv,100)
```

```
est
marginal1value<-mnntsmarginalgeneraldimension(cpars=est$cestimates,
M=Mbiv,R=Rbiv,marginal=1,theta=matrix(c(pi/2),nrow=1,ncol=1))
marginal1value

# A bivariate marginal from a trivariate joint

set.seed(200)
Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
data<-WindDirectionsTrivariate
est<-mnntsmanifoldnewtonestimation(data,Mtriv,Rtriv,100)
est
marginal12value<-mnntsmarginalgeneraldimension(cpars=est$cestimates,
M=Mtriv,R=Rtriv,marginal=c(1,2),theta=matrix(c(pi/4,pi/2),nrow=1,ncol=2))
marginal12value
```

---

mnntsparametersunderindependenceunivariate *Marginal Density Function at a Vector of Fixed Values*

Description

Computes the vector of c parameters of an MNNTS density from the vectors of c parameters of its independent marginals

Usage

```
mnntsparametersunderindependenceunivariate(data,R,Mvector,cparlist)
```

Arguments

`data`	Matrix of angles in radians (with R columns)
`R`	Number of dimensions
`Mvector`	Vector of M parameters. A nonnegative integer number for each of the R components of the vector
`cparlist`	A list in which each element is a matrix containing the information of the vector of c parameters for each independent marginal component

Value

`cestimates`	Matrix of prod(M+1)*(R+1). The first R columns are the parameter number, and the last column is the c parameter's estimators
`loglik`	Log-likelihood value
`AIC`	Value of Akaike's Information Criterion
`BIC`	Value of Bayesian Information Criterion

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

References

Fernandez-Duran and J. J. and Gregorio-Dominguez and M. M (2023). Multivariate Nonnegative Trigonometric Sums Distributions for High-Dimensional Multivariate Circular Data, arXiv preprint arXiv:2301.03643v2

Examples

```
# Bivariate MNNTS density from independent marginals

set.seed(200)
Mbiv<-c(2,3)
Rbiv<-length(Mbiv)
data(Nest)
data<-Nest*(pi/180)
est1<-nntsmanifoldnewtonestimation(data[,1],Mbiv[1])
est1
est2<-nntsmanifoldnewtonestimation(data[,2],Mbiv[2])
est2
est12independent<-mnntsparametersunderindependenceunivariate(data,R=Rbiv,
Mvector=Mbiv,cparlist=list(est1,est2))
est12independent

# Trivariate MNNTS density from independent marginals

set.seed(200)
Mtriv<-c(2,3,3)
Rtriv<-length(Mtriv)
data(WindDirectionsTrivariate)
data<-WindDirectionsTrivariate
est1<-nntsmanifoldnewtonestimation(data[,1],Mtriv[1],70)
est1
est2<-nntsmanifoldnewtonestimation(data[,2],Mtriv[2],70)
est2
est3<-nntsmanifoldnewtonestimation(data[,3],Mtriv[3],70)
est3
est123independent<-mnntsparametersunderindependenceunivariate(data,R=Rtriv,
Mvector=Mtriv,cparlist=list(est1,est2,est3))
est123independent
```

Nest — *Nest orientations and creek directions*

Description

Orientation of nests of 50 noisy scrub birds (theta) along the bank of a creek bed, together with the corresponding directions (phi) of creek flow at the nearest point to the nest.

Usage

```
data(Nest)
```

Format

Orientation of 50 nests (vectors)

Source

Data supplied by Dr. Graham Smith

References

N.I. Fisher (1993) Statistical analysis of circular data. Cambridge University Press.

WindDirectionsTrivariate *Wind directions*

Description

Wind directions registered at the monitoring stations of San Agustin located in the north, Pedregal in the southwest, and Hangares in the southeast of the Mexico Central Valley's at 14:00 on days between January 1, 1993 and February 29, 2000. There are a total of 1,682 observations

Usage

```
data(WindDirectionsTrivariate)
```

Format

Three columns of angles in radians

Source

Mexico Central Valleys pollution monitoring network. RAMA SIMAT (Red Automatica de Monitoreo Ambiental)

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